Task 2: Sudoku

Sullivan Dolkan Kuralski likes puzzle challenges. One of them is Sudoku. The task is to place digits in a square with 9×9 fields such that each field has one digit from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and furthermore, in any row, column or quadrant (which is one of the nine 3×3 subsquares obtained by dividing the field evenly into 3×3 subsquares) the digits 1 to 9 occur exactly once. Below is an example of a Sudoku and also an example of a field a in a Sudoku with its row r, its column c and another field b with its quadrant q; so the entries marked with a and r are the row of a, the entries marked with a and c are the column of a and the entries marked with b and q are the quadrant of b.

```
1 2 3  4 5 6  7 8 9      . . .  . . .  . c .
7 8 9  1 2 3  4 5 6      r r r  r r r  r a r
4 5 6  7 8 9  1 2 3      . . .  . . .  . c .

9 1 2  3 4 5  6 7 8      . . .  q q q  . c .
6 7 8  9 1 2  3 4 5      . . .  q q b  . c .
3 4 5  6 7 8  9 1 2      . . .  q q q  . c .

8 9 1  2 3 4  5 6 7      . . .  . . .  . c .
5 6 7  8 9 1  2 3 4      . . .  . . .  . c .
2 3 4  5 6 7  8 9 1      . . .  . . .  . c .
```

The Sudoku game provides a 9×9 square with some empty fields. The player needs to recover the digits in those empty fields. As Sullivan became a well-known expert for Sudokus, an international newspaper offered him a job: When he creates Sudoku tasks for each of its editions, they would pay him a honorarium. However, the Sudoku tasks which he makes should satisfy certain conditions:

- They should have as many empty fields as possible (here indicated as a 0 and in the printed newspaper just left blank);
- They should have only a single solution (so that the solution of the newspaper published the next day will match what the readers did in the case that they did it right);
- They should be human-solvable, that is, at every time there should be at least one free field which can be filled in by human with a unique digit.

The third condition is a bit fuzzy and when Sullivan enquired on what it meant precisely, the newspaper hesitated a bit but eventually came up with the following answer:

- At every stage of solving the Sudoku task, there must be a still free field so that every digit except the correct one is already filled-in at some other field in the column, the row or the quadrant of this free field; here "filled-in" includes the given digits and the digits filled-in in previous steps of making the solution.

Below gives an example that satisfies the human-solvable condition. In the example, all fields with zero are empty fields. The field "a" is forced to be "1" since the entries in the column, the row and the quadrant contain all possible digits except "1":

```
. . 0  . . .  . . .
. . 2  . . .  . . .
. . 0  . . .  . . .

4 5 a  0 0 0  6 8 9
6 7 0  . . .  . . .
0 0 0  . . .  . . .

. . 0  . . .  . . .
. . 3  . . .  . . .
. . 0  . . .  . . .
```

Sullivan knew that he could make such Sudokus, but it would need some time; so he did not accept the job right away. Instead he asked his friend Claude Pastel Procal for help and Claude answered: "Why don't you use computers? Sometimes simple algorithms can solve difficult problems." So your task is now to write a program which makes the Sudoku tasks for Sullivan according to the conditions above. Furthermore, his boss at the newspaper does not want that all Sudokus have the same solution; therefore he will provide Sullivan with ideas where some of the 1s have to be in the final Sudoku. So Sullivan and his boss agreed on the following protocol:

- The boss provides Sullivan with 81 numbers: These 81 numbers are either 0 or 1 and contain suggestions of the boss where should be a 1; note that the boss might only suggest some and not all of the positions of 1s.
- Sullivan should deliver 81 numbers representing the entries of the Sudoku task: a 0 indicates a free field and an entry from 1 up to 9 indicates a prefilled field. The Sudoku task should be human solvable, it should have as many free fields as possible and the solution of the Sudoku should have a 1 at every place where the boss had suggested that there is a 1.

The boss will ask his IT department to make a computer program which checks the requirements he has put on Sullivan and he will only pay for those Sudoku tasks which meet the requirements. So your computer program should produce Sudoku tasks which meet the above specifications.

Input Format

Your program should read the input from standard input. The input consists of 81 numbers. These are arranged row by row by the format of a Sudoku and those which are 1 should have a 1 in the final solution of the Sudoku. Below is an example **input**:

```
1 0 0  0 0 0  0 0 0
0 0 0  1 0 0  0 0 0
0 0 0  0 0 0  1 0 0

0 1 0  0 0 0  0 0 0
0 0 0  0 1 0  0 0 0
0 0 0  0 0 0  0 1 0

0 0 1  0 0 0  0 0 0
0 0 0  0 0 1  0 0 0
0 0 0  0 0 0  0 0 1
```

Output Format

Your program should write 81 numbers onto standard output satisfying the specifications given above. These 81 numbers are a valid Sudoku task which has to meet the positions of the 1s given by the Boss. The entries 1 to 9 in the 81 numbers correspond to the prefilled positions with the corresponding numbers in the Sudoku task. Here a possible **output** which matches the above input, note that the output is not unique.

```
0 0 3  4 5 6  7 8 9
0 8 9  1 2 3  4 5 6
0 5 6  7 8 9  1 2 3

9 1 2  3 4 5  6 7 8
6 7 8  9 1 2  3 4 5
3 4 5  6 7 8  9 1 2

8 9 1  2 3 4  0 6 7
5 6 7  8 9 1  2 3 4
0 3 4  5 6 7  8 9 1
```

The output leaves 6 fields free and would score 5 marks according to the assessment table below. It is permitted that some of the 1s of the boss are left blanc; however, when the solution is made, there must be a 1 which goes into this position.

Assessment

For assessment, the program receives inputs with 1, 3, 5, 7 and 9 preassigned 1s, respectively; the maximum execution time for each subtask is 1.0s. Each subtask can be scored up to 20 marks. It is evaluated as follows: If the solution is not human solvable or if it does not match the predefined 1s then it scores 0 marks; otherwise the score is at least 3 and each three free fields give 1 extra mark, but not more than 20 marks. The following table summarises the marking scheme.

Not human solvable	0 marks
Not matching the 1s	0 marks
Correct and 0–2 free fields	3 marks
Correct and 3–5 free fields	4 marks
Correct and 6–8 free fields	5 marks
Correct and 9–11 free fields	6 marks
Correct and 12–14 free fields	7 marks
Correct and 15–17 free fields	8 marks
Correct and 18–20 free fields	9 marks
Correct and 21–23 free fields	10 marks
Correct and 24–26 free fields	11 marks
Correct and 27–29 free fields	12 marks
Correct and 30–32 free fields	13 marks
Correct and 33–35 free fields	14 marks
Correct and 36–38 free fields	15 marks
Correct and 39–41 free fields	16 marks
Correct and 42–44 free fields	17 marks
Correct and 45–47 free fields	18 marks
Correct and 48–50 free fields	19 marks
Correct and 51–81 free fields	20 marks